DAMPING OF LONG-WAVELENGTH KINETIC ALFVÉN FLUCTUATIONS: LINEAR THEORY

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Abstract

The full electromagnetic linear dispersion equation for kinetic Alfvén fluctuations in a homogeneous, isotropic, collisionless, Maxwellian electron-proton plasma is solved numerically in the long wavelength limit. At propagation sufficiently oblique to the background magnetic field $\mathbf{B}_o$, the solutions are summarized by an analytic expression for the damping rate of such modes which scales as $k_\perp^2 k_\parallel$ where the subscripts denote directions relative to $\mathbf{B}_o$. This damping progressively (although not monotonically) increases with increasing electron and proton β, corresponding to four distinct damping regimes: nonresonant, electron Landau, proton Landau, and proton transit-time damping.

1. Introduction

This manuscript addresses the damping of long-wavelength Alfvénic fluctuations propagating obliquely to a background magnetic field $\mathbf{B}_o$ in a homogeneous, isotropic, collisionless plasma. We assume an electron-proton plasma in which the velocity distributions are Maxwellian. We use numerical solutions of the linear dispersion equation for arbitrary directions of propagation [*Gary*, 1993] to obtain an analytic expression for such damping as a function of dimensionless plasma parameters and wavevector components.

Alfvén-cyclotron fluctuations are left-hand circularly polarized at propagation parallel to the background magnetic field $\mathbf{B}_o$, satisfy the dispersion relation $\omega_r = k_\parallel v_A$ at long wavelengths, and are relatively incompressible at most angles of propagation. Here the subscripts $\parallel$ and $\perp$ denote directions relative to $\mathbf{B}_o$ and v_A represents the Alfvén speed. At $\mathbf{k} \times \mathbf{B}_o = 0$ the fluctuating fields of an Alfvén-cyclotron mode are perpendicular to $\mathbf{B}_o$ and in linear theory the cyclotron resonance provides the only important wave-particle interaction. At sufficiently large values of $k_\parallel c/\omega_p$ (where c/ω_p represents the proton inertial length), such fluctuations develop a strong ion cyclotron resonance, so that proton cyclotron damping becomes the dominant dissipation mechanism. The wavenumber at onset of this damping is typically of order $k_{\|}c/\omega_p \sim 1$; e.g. $k_{\|}c/\omega_p \simeq 0.5$ at $\beta_p = 0.10$ [*Gary and Borovsky*, 2004].

As the wavevector $\mathbf{k}$ becomes more oblique to $\mathbf{B}_o$, the properties of Alfvénic fluctuations change; at sufficiently large values of θ the fluctuations are often called "kinetic Alfvén waves" [*Hollweg*, 1999, and references therein]. The nonzero $k_\perp$ admits nonzero $\delta E_{\|}$ and $\delta B_{\|}$ via the Landau resonance at $\omega_r = k_{\|}v_{\|}$. The parallel fluctuating electric field implies Landau damping of the wave, and the compressible magnetic field gives rise to transit time damping [*Stix*, 1992].

Our concern here is the damping of kinetic Alfvén fluctuations at $k_\perp \neq 0$ and $k_{\|}c/\omega_p << 1$, where the primary dissipation mechanisms are via the Landau resonances. Previous analyses of such damping using dispersion theory derived from the linear Vlasov equation have been carried out by *Stefant* [1970], *Lysak and Lotko* [1996], and *Leamon et al.* [1999]. *Gary and Borovsky* [2004] showed that, for sufficiently oblique propagation, for sufficiently long wavelengths, and for a relatively broad range of plasma parameters, this damping in an electron-proton plasma can be expressed in the form

$$\frac{\gamma}{\Omega_p} = -A\left(\frac{m_e}{m_p}\right)^{1/2}\beta_e^{1/2}\left(\frac{k_\perp c}{\omega_p}\right)^2\frac{k_{\|}c}{\omega_p} \tag{1}$$

Here A is a parameter which is a function of β_e and T_e/T_p; Ω_j stands for the jth species cyclotron frequency, ω_j represents the jth species plasma frequency, the jth species thermal speed is $v_j \equiv \sqrt{k_BT_j/m_j}$, and $\beta_j \equiv 8\pi n_j k_B T_j/B_o^2$. This report describes an evaluation of the parameter A and provides an interpretation of that evaluation.

2. Linear theory

We carried out numerical evaluations of the full electromagnetic dispersion equation for kinetic Alfvén fluctuations in the long wavelength limit. For an electron-proton plasma with Maxwellian velocity distributions, the analytic expression for the dispersion equation is given, for example, in *Gary* [1993]. Here $m_p/m_e = 1836$; sample calculations show the results illustrated here are independent of v_A/c as long as $\Omega_e^2/\omega_e^2 = m_p/m_e(v_A/c)^2 << 1$. The Landau resonance factor for the jth species is defined as $\zeta_j \equiv \omega/\sqrt{2}|k_{\|}|v_j$.

In the long wavelength limit, we found γ/Ω_p to approximately satisfy the $(k_\perp^2 c/\omega_p)^2 k_{\|}c/\omega_p$ wavenumber dependence of Equation (1) for sufficiently large values of θ, the angle of propagation relative to $\mathbf{B}_o$. Here "sufficiently large" is a function of β_e;

Figure 1 shows how, at $\beta_e = 10^{-4}$, Equation (1) is appropriate for $30^o \lesssim \theta < 90^o$, whereas on the range $0.001 \leq \beta_e \leq 1.0$, Equation (1) is applicable only over $60^o \lesssim \theta < 90^o$.

Kinetic Alfvén fluctuations at long wavelengths well satisfy the dispersion relation

$$\omega_r = k_\parallel v_A$$

So the Landau resonance factors ζ_e and ζ_p are inverse functions of β_e and β_p, respectively, as illustrated in Figure 2a. For a species j and a particular mode, if $\zeta_j >> 1$, the jth species is non-Landau-resonant and that species should make no contribution to the damping rate. As ζ_j becomes less than about 3, the parallel phase speed $\omega_r/k_\parallel$ approaches the tail of the reduced velocity distribution $f_j(v_\parallel)$ and there is the onset of damping due to the Landau wave-particle resonance. As ζ_j decreases further, the parallel phase speed moves deeper into the thermal part of the reduced velocity distribution, so that, other things being equal, the mode in question interacts with more particles and the damping increases monotonically. Thus Figure 2a suggests that, as β_e increases, kinetic Alfvén waves should experience the onset of electron Landau damping at $\beta_e << 1$, and that the proton Landau resonance should provide additional damping at $\beta_p \gtrsim 1$ [*Borovsky and Gary*, 2008].

But other things are not always equal, and the suggestion of a relatively monotonic increase in damping with β_e is not substantiated by detailed linear theory computations. Anticipating the results of Figure 3, Figure 2b illustrates the Landau resonance factors as functions of β_e for both kinetic Alfvén and ion acoustic fluctuations. This panels shows a crossing of the resonance factors for the two modes at $\beta_e \simeq 2.5$; the interaction of the two modes in complex frequency space suggests that additional physics arises near this β value.

Figure 3 shows the coefficient A as a function β_e for $\theta = 60^o$. The figure shows four distinct parameter regimes. If $v_p << v_e << v_A$ ($\beta_e << 2m_e/m_p$), $\omega_r/k_\parallel$ is much greater than the thermal speed of either species, kinetic Alfvén fluctuations are non-(Landau) resonant (i.e. $|\zeta_p| >> |\zeta_e| >> 1$), and damping is very weak; in this regime these fluctuations are termed "inertial Alfvén waves" [*Lysak and Lotko*, 1996]. As β_e increases, $\omega_r/k_\parallel$ approaches the electron thermal speed, $|\zeta_e|$ approaches unity (as indicated by Figure 2a), and electron Landau damping becomes significant. The parameter regime in which this mechanism dominates damping is $v_p < v_A \lesssim v_e$, that is $2m_e/m_p \lesssim \beta_e << 2T_e/T_p$. In this regime, the A factor is approximately constant, as noted by *Gary and Borovsky* [2004], although at $\beta_e = 0.10$ we obtain $A = 0.54$, which is larger than the 0.35 factor computed by *Gary and Borovsky* [2004] for a nonvanishing value of $k_\parallel$.

As the species β further increase, electron Landau damping persists, but the continuing decrease in $|\zeta_p|$ for kinetic Alfvén fluctuations implies the onset of additional damping due to the proton Landau resonance. This increase was predicted by *Stefant* [1970], who constructed an approximate solution to the linear dispersion equation for kinetic Alfvén fluctuations which included the effects of $\delta E_{\|}$ and Landau damping, but assumed $\delta B_{\|} = 0$ and therefore ignored transit-time damping. *Stefant* [1970] predicted a maximum in the magnitude of the damping rate as a function of β_e, interpreting this as resulting from a matching between the phase speeds of the Alfvén and ion acoustic fluctuations.

Our numerical solutions for the damping rate of kinetic Alfvén fluctuations in the long-wavelength limit, illustrated in Figure 3, also exhibit a maximum as a function of β_e. This maximum is a function of T_e/T_p, also illustrated in Figure 3. At $T_e/T_p = 10$, $\beta_p = 0.25$ and $\theta = 60^o$, the parallel phase speeds of the kinetic Alfvén and ion acoustic modes in the long wavelength limit are the same; that is, $\zeta_p = 2.0$. The *Stefant* [1970] interpretation implies that this should be correspond to the maximum value of the A factor, in qualitative agreement with the results of Figure 3.

Further support for the phase-speed-matching interpretation is given by the character of the peak in the A-vs-β_e curve. This relative maximum is distinct at $T_e/T_p = 10$ where ion acoustic wave damping is weak, becomes less distinct as ion acoustic damping increases at $T_e/T_p = 1$ [*Stefant*, 1970], and disappears completely at $T_e/T_p = 0.10$ where proton Landau damping is so strong that ion acoustic modes cannot propagate. Figure 4, which plots the kinetic Alfvén fluctuation damping rate and the associated value of $|\delta E_{\|}|^2/|\delta \mathbf{E}|^2$ as functions of β_e, shows that the relative maximum for damping of this mode coincides with a relative maximum of this fluctuating electric field ratio. In other words, when the $\omega_r/k_{\|}$ of the kinetic Alfvén and ion acoustic modes match, the relatively strong fluctuating parallel electric field of the latter mode couples to the same field component of the former mode, enhancing the $|\delta E_{\|}|$ and thereby enhancing the damping of the kinetic Alfvén fluctuations.

If $v_A < v_p$ (i.e., $\beta_p > 2$), Alfvén fluctuation damping increases monotonically with β. This is very different from the damping rate in the electron Landau damping regime, where A is relatively independent of changes in β_e. To examine a possible reason for this difference, we return to *Gary and Borovsky* [2004] and evalute the efficacy ratios R_j which provide a measure of the relative effectiveness of transit-time damping (which depends on $\delta B_{\|}$) versus Landau damping (which is a function of $\delta E_{\|}$). From Equation (7) of *Gary*

Table 1: Kinetic Alfvén fluctuation damping regimes

		Primary damping
$\beta_e << 2m_e/m_p$	$v_e << v_A$	Nonresonant
$2m_e/m_p \lesssim \beta_e << 2T_e/T_p$	$v_p << v_A \lesssim v_e$	Electron Landau
$\beta_p \lesssim 2$	$v_p \lesssim v_A$	Proton Landau
$2 < \beta_p$	$v_A < v_p$	Proton transit-time

and Borovsky [2004],

$$\frac{R_e}{R_p} = \frac{T_e^2}{T_p^2} \tag{2}$$

for an electron-proton plasma, so that if $T_e = T_p$, we need consider only R_e.

Figure 5a shows the electron efficacy ratio as a function of θ for kinetic Alfvén fluctuations in the long wavelength limit for $T_e/T_p = 1$. For $\beta_e \lesssim 1$, R_e is much less than 1 indicating that Landau damping dominates transit-time damping for both electrons and protons at all angles of propagation. As β_e becomes greater than unity, R_e at quasi-perpendicular propagation grows, so that, at sufficiently large β_e, transit-time damping dominates Landau damping for both species. This is illustrated in Figure 5b which shows that R_e at $\theta = 60^o$ increases monotonically as a function of β_e.

Figure 6 shows the electron and proton efficacy ratios as functions of θ for kinetic Alfvén fluctuations in the long wavelength limit for three different values of T_e/T_p at $\beta_p = 1$. As Equation (2) suggests, an increase in T_e/T_p implies an increasing efficacy of electron transit-time damping, at both quasi-parallel and quasi-perpendicular propagation, whereas a decreasing T_e/T_p leads to an increased role of proton transit-time damping.

3. Summary

We have numerically solved the full linear dispersion equation for kinetic Alfvén fluctuations in a homogeneous, isotropic, collisionless plasma. At sufficiently long wavelengths, and at sufficiently large angles of propagation relative to $\mathbf{B}_o$, the damping of such modes can be expressed as Equation (1) with coefficient A that depends only on β_e and T_e/T_p. Our numerical solutions show that A and the associated damping generally increase as β_e increases through the four regimes summarized in Table 1.

Equation (1) can be used in fluid models of long-wavelength kinetic Alfvén fluctuations to accurately include the dissipation due to the Landau resonances of such fluctuations. In particular, we suggest that this equation provides a simple method for including the consequences of kinetic Alfvén fluctuation dissipation in MHD turbulence models [*Borovsky and Gary*, 2008]. The damping of magnetosonic fluctuations is different from Equation (1) and must be derived separately.

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Figure Captions

Figure 1. The coefficient A of Equation (1) as a function of propagation angle θ for five different values of β_e. For all figures presented here, this coefficient is derived from

numerical solutions of the full linear dispersion equation for kinetic Alfvén fluctuations in the limit of kc/ω_p approaching zero using $m_p/m_e = 1836$ and $v_A/c = 1.0 \times 10^{-4}$. Here the dotted line corresponds to $\beta_e = 1.0$, the long-dash-short-dash line to $\beta_e = 0.10$, the solid line to $\beta_e = 0.01$, the uniformly dashed line to $\beta_e = 0.001$, and the line of squares to $\beta_e = 0.0001$. Here $T_e/T_p = 1.0$.

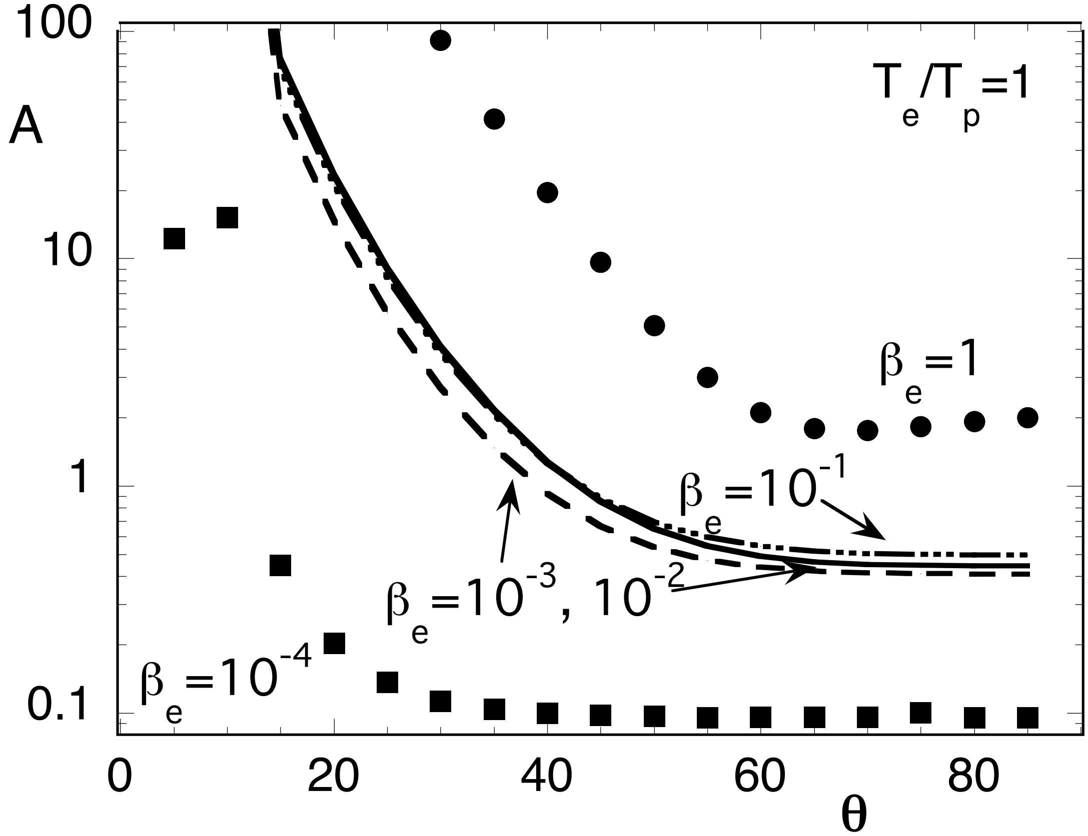
Figure 2. The Landau resonance factors for fluctuations in the long-wavelength limit at $\theta = 60^o$ for electrons (ζ_e) and protons (ζ_p) as functions of β_e. (a) Resonance factors for kinetic Alfvén waves; here $T_e/T_p = 1$ and 10 as labeled. (b) At $T_e/T_p = 10$, resonance factors for kinetic Alfvén waves (ζ_p is the solid line and ζ_e is the line of solid dots) and for ion acoustic waves (ζ_p is the dashed line and ζ_e is the line of open circles).

Figure 3. The coefficient A of Equation (1) for kinetic Alfvén fluctuations in the long-wavelength limit as a function of β_e at $\theta = 60^o$ for three values of T_e/T_p as labeled.

Figure 4. The damping rate and associated value of $|\delta E_{\parallel}|^2/|\delta \mathbf{E}|^2$ of kinetic Alfvén fluctuations at $kc/\omega_p = 0.01$, $\theta = 60^o$ and $T_e/T_p = 10$ as functions of β_e.

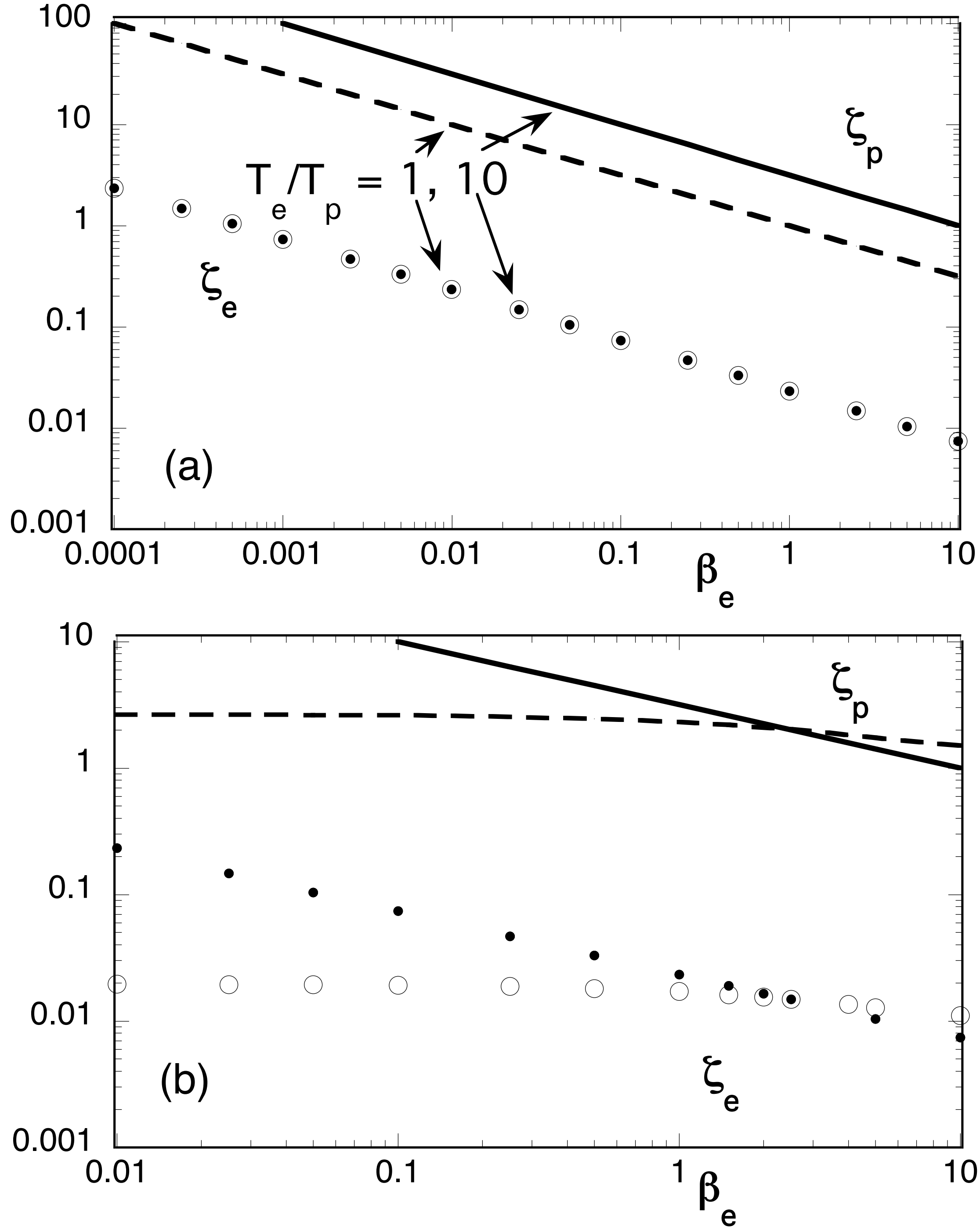
Figure 5. The electron efficacy ratio R_e for kinetic Alfvén fluctuations in the long wavelength limit. (a) R_e as a function of the propagation angle θ for four different values of β_e as labeled. (b) R_e at $\theta = 60^o$ as a function of β_e. Here $T_e/T_p = 1.0$.

Figure 6. The species efficacy ratios R_j for kinetic Alfvén fluctuations in the long wavelength limit as functions of the propagation angle θ. Here $\beta_p = 1.0$, and the three different values of T_e/T_p are as labeled. Here R_p is represented by the dashed line for $T_e/T_p = 2.0$, by the solid line for $T_e/T_p = 1.0$, and by the long-dash-short-dashed line for $T_e/T_p = 0.5$. Here R_e is represented by the line of diamonds for $T_e/T_p = 2.0$, by the line of squares for $T_e/T_p = 1.0$, and by the line of dots for $T_e/T_p = 0.5$.



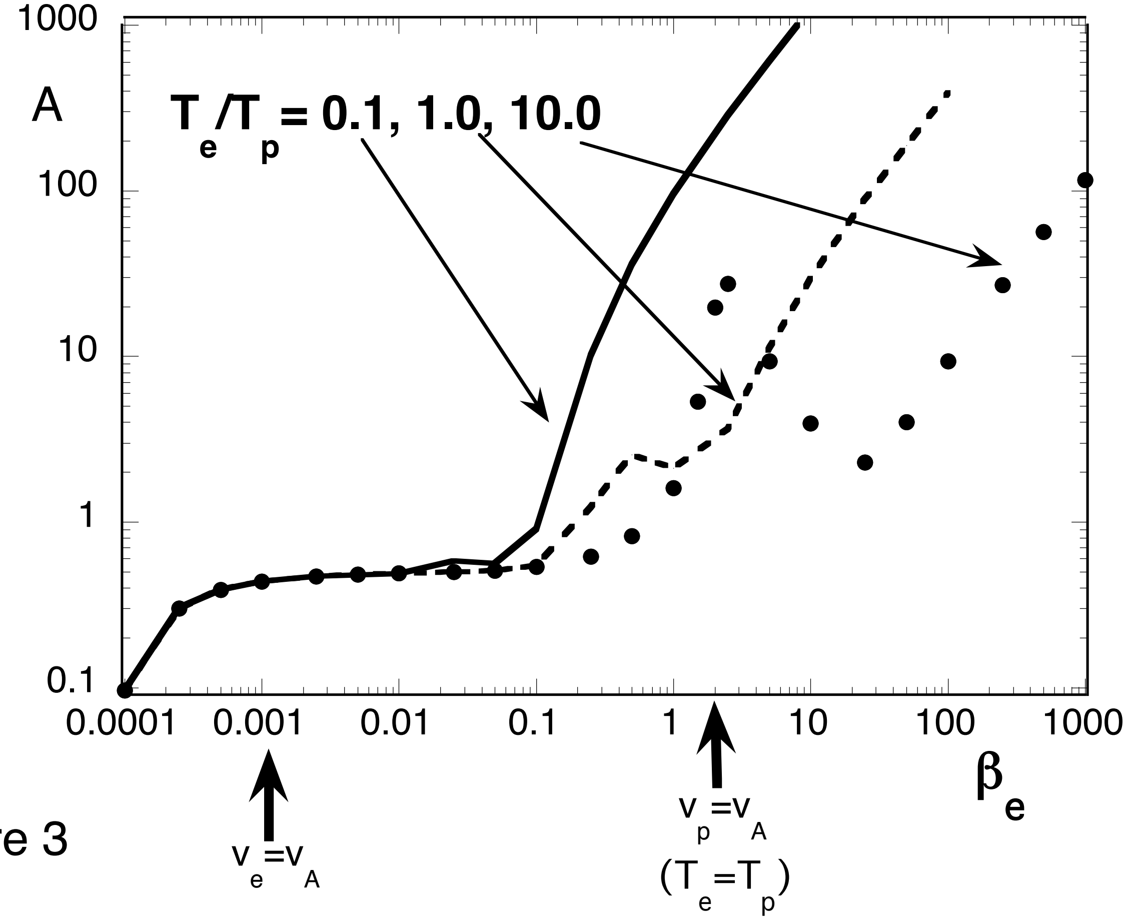
100
10
1
0.1
A
0
20
40
60
80
θ
$T_e/T_p=1$
$\beta_e=1$
$\beta_e=10^{-1}$
$\beta_e=10^{-3},\ 10^{-2}$
$\beta_e=10^{-4}$

Figure 1



100
10
1
0.1
0.01
0.001
0.0001
0.001
0.01
0.1
1
10
β
e
ζ
p
ζ
e
T
e
/T
p
= 1, 10
(a)
10
1
0.1
0.01
0.001
0.01
0.1
1
10
β
e
ζ
p
ζ
e
(b)

Figure 2



1000
100
10
1
0.1
A
T_e/T_p = 0.1, 1.0, 10.0
0.0001
0.001
0.01
0.1
1
10
100
1000
β_e
$v_e=v_A$
$v_p=v_A$
($T_e=T_p$)

Figure 3

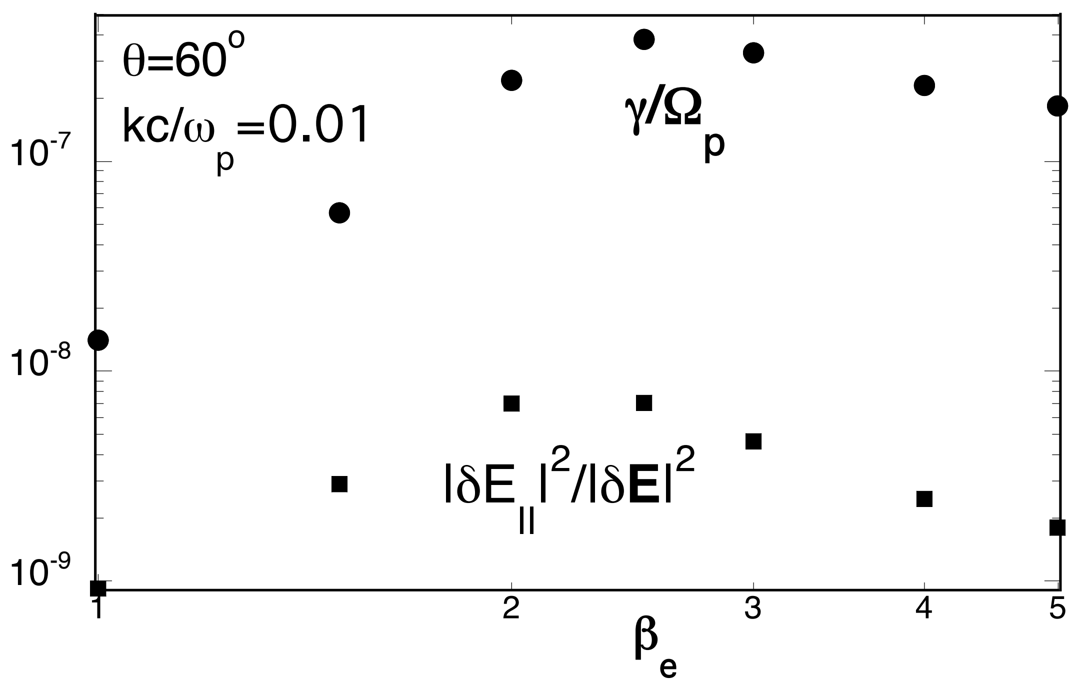
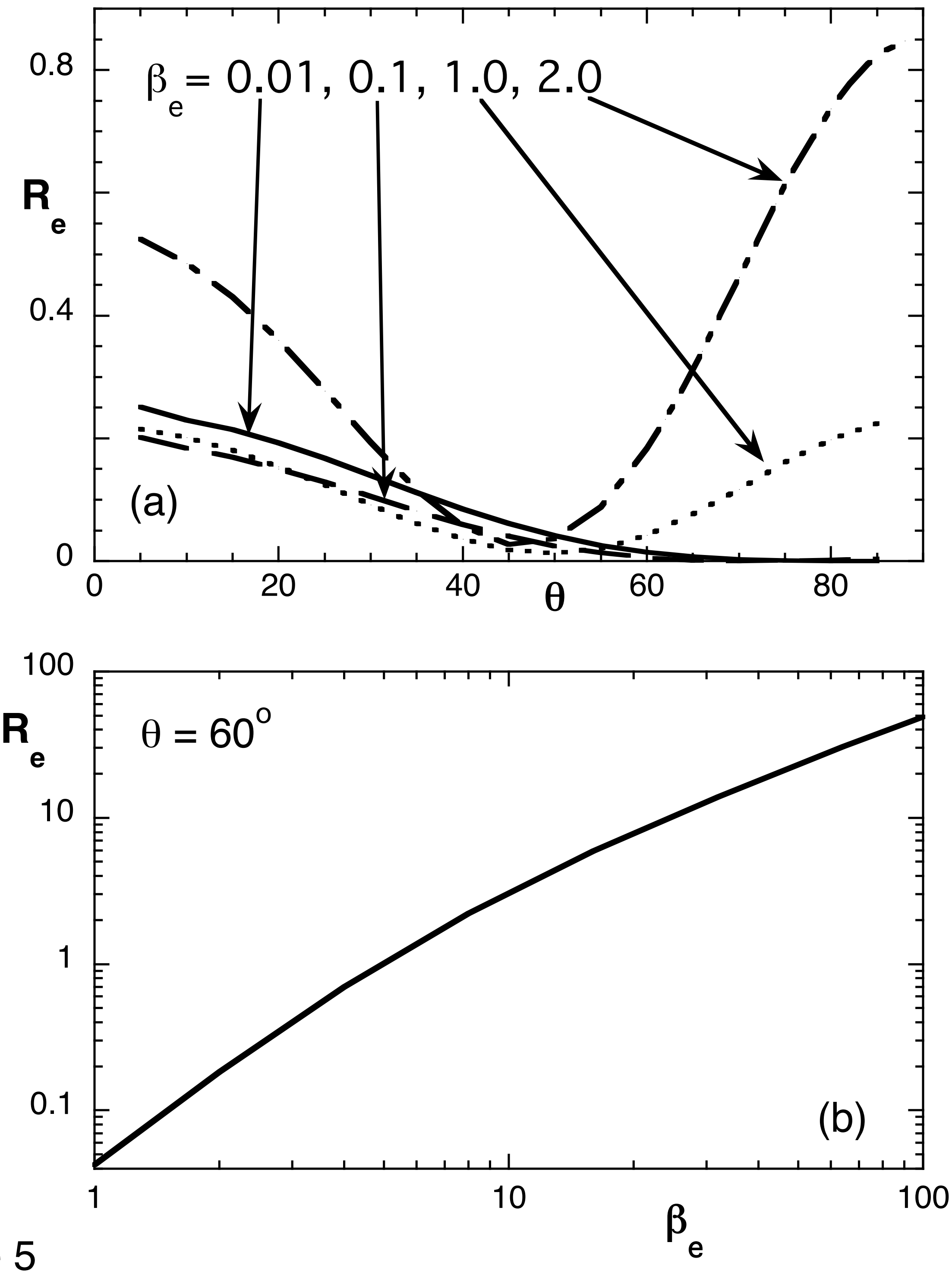


Figure 4



β_e= 0.01, 0.1, 1.0, 2.0
R_e
0.8
0.4
0
(a)
0
20
40
θ
60
80
100
R_e
$\theta = 60^o$
10
1
0.1
(b)
1
10
100
β_e

Figure 5

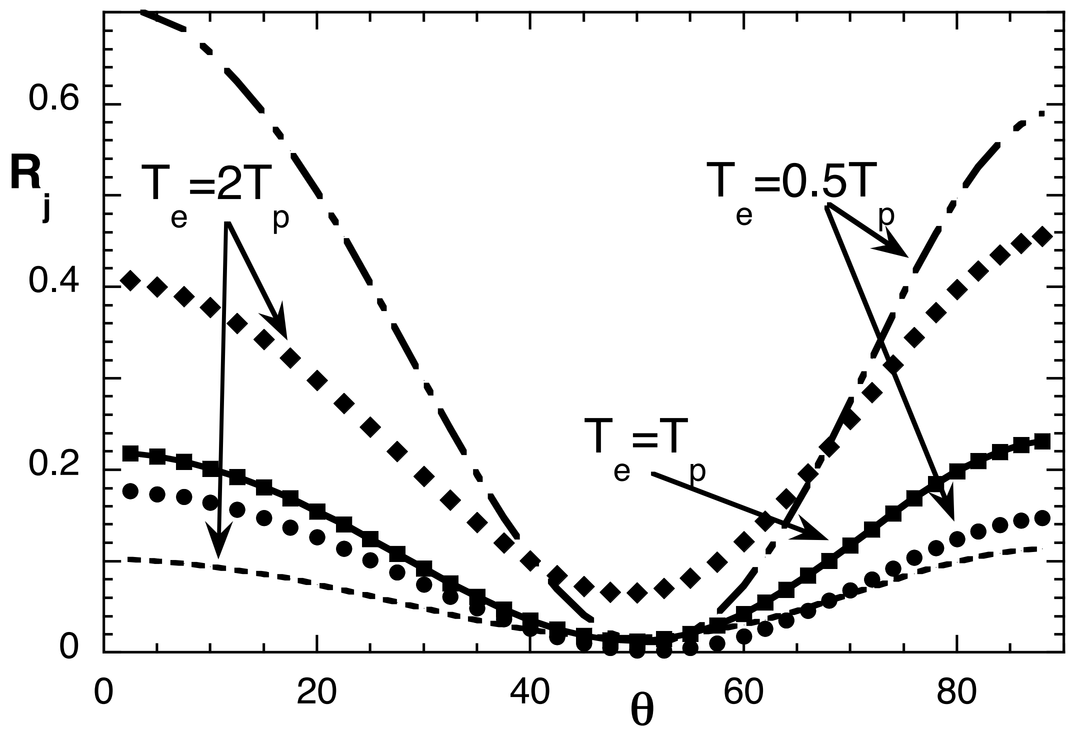


Figure 6